HYDRO STATICS

FLUID ACTION ON SURFACES

Fluid Action on Surfaces

Plane Surfaces

If the pressure over a plane area is uniform, as in the case of a horizontal surface submerged in a liquid or a plane surface inside a gas chamber, the total hydrostatic force (or total pressure) is given by:

$$F = pA$$

where p is the uniform pressure and A is the area.

In the case of an inclined or vertical plane submerged in a liquid, the total pressure can be found by the following formula:

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Consider the plane surface shown inclined at an angle θ with the horizontal. To get the total force F, consider a differential element of area dA. Since this element is horizontal the pressure is uniform over this area, then;

$$dF = pdA$$

Where $p = \gamma h = \gamma\, y\, sin\ \theta$

$$dF = \gamma\, y \sin\theta\, dA$$

$$\int dF = \int \gamma\, y \sin\theta\, dA$$

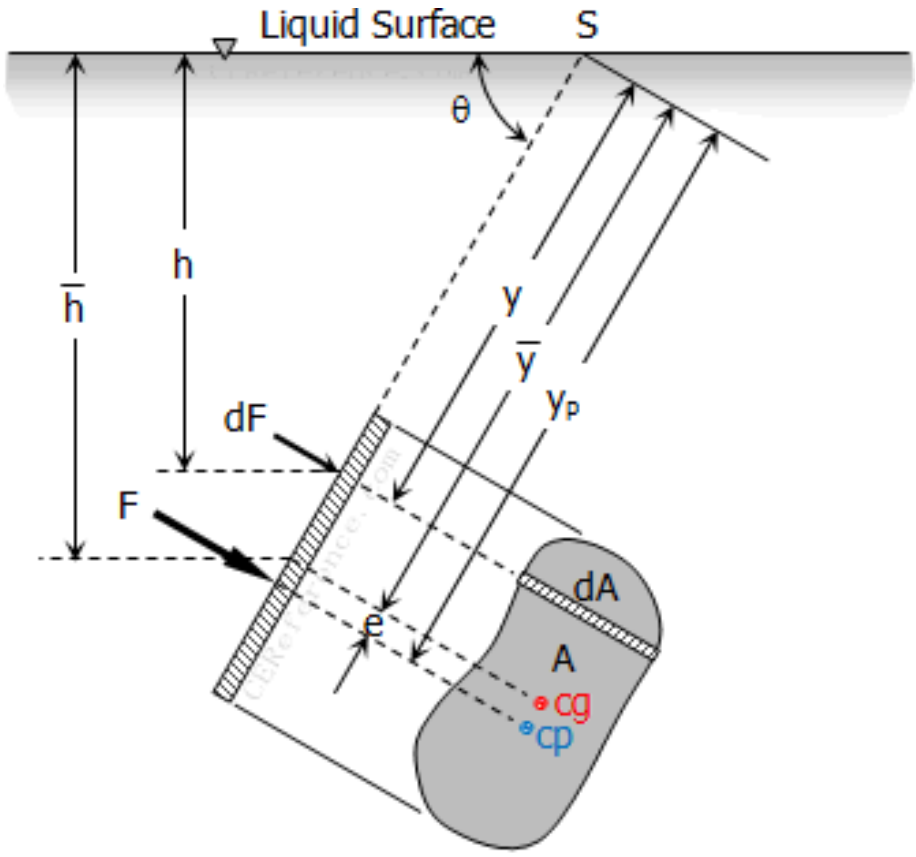

Figure 03: Forces on an inclined plane surface

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Plane Surfaces

From calculus, $\int ydA = A\bar{y}$

$$F = \gamma \, sin\theta \, A\bar{y}$$
$$F = \gamma \, (\bar{y}sin\theta)A$$
$$F = \gamma\hbar A$$

Since γħ is the unit pressure at the centroid of the plane area, p_{cg}, the formula may also be expressed as:

$$F = p_{cg}A$$

$F = \gamma\hbar A$ is convenient to use if the plane is submerged in a single liquid and without gage pressure at the surface of the liquid. However, if the plane is submerged under layers of different liquids or if the gage pressure at the liquid surface is not zero, $F = p_{cg}A$ is easier to apply.

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Location of F (y_p):

In the Figure, taking moment of force about S, (the intersection of the prolongation of the plane area and the liquid surface),

$$F y_p = \int y dF$$

Where $dF = \gamma\, y \sin\theta\, dA$

$$F = \gamma\, sin\theta\, A\bar{y}$$

$$\gamma sin\theta A\bar{y} y_p = \int y(\gamma y sin\theta dA)$$

$$\gamma sin\theta A\bar{y} y_p = \gamma sin\theta \int y^2 dA$$

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From calculus, $\int y^2\, dA = I_s$ (moment of inertia about S)

$$A\bar{y}y_p = I_s$$

$$y_p = \frac{I_s}{A\bar{y}}$$

By transfer formula of moment of inertia:

$$I_s = I_g + A\bar{y}^2$$

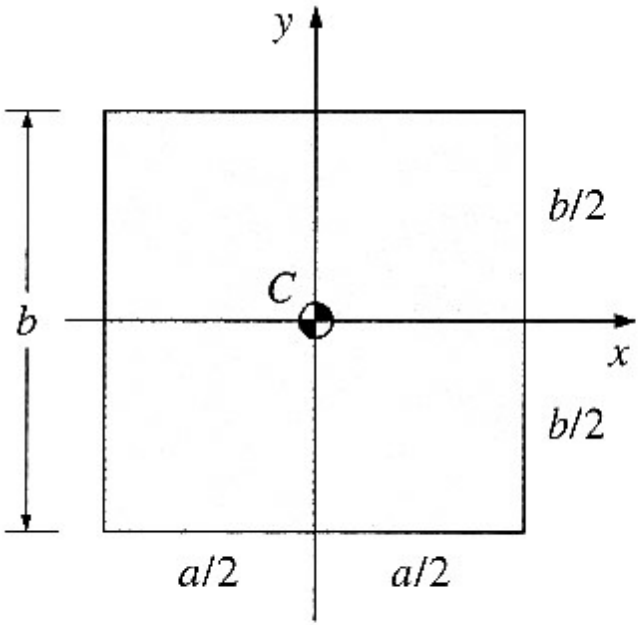
$$y_p = \frac{I_g + A\bar{y}^2}{A\bar{y}}$$

$$y_p = \bar{y} + \frac{I_g}{A\bar{y}}$$

$$y_p = \bar{y} + e$$

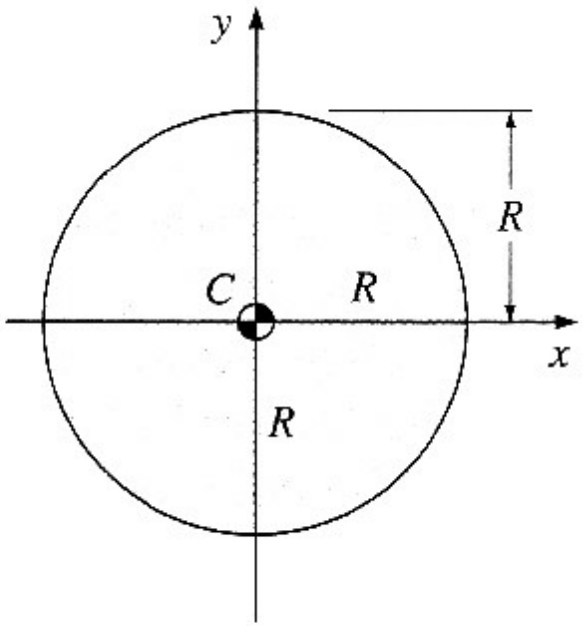
Fluid Action on Surfaces

Plane Surfaces Properties



$A = ab,\ I_{xx,\,C} = ab^3/12$

(*a*) Rectangle



$A = \pi R^2,\ I_{xx,\,C} = \pi R^4/4$

(*b*) Circle

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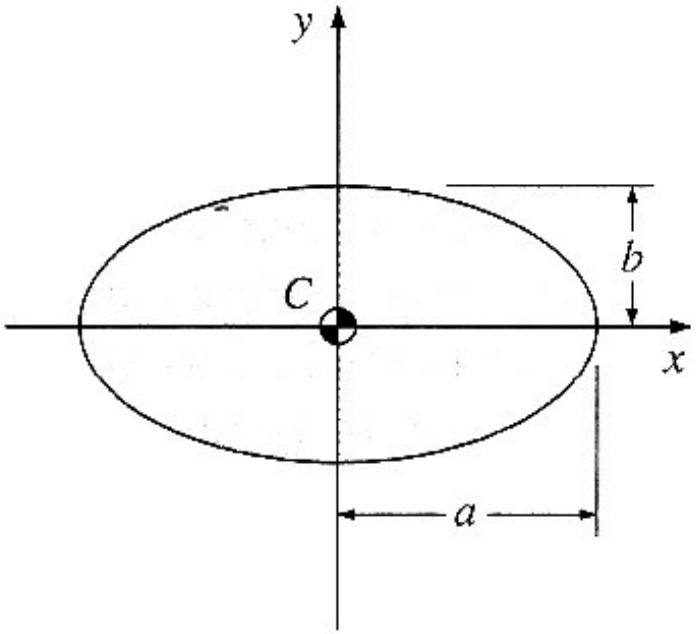
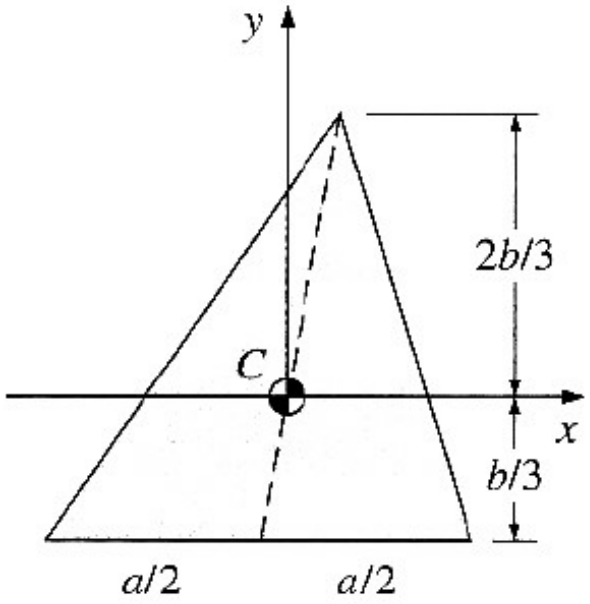
Plane Surfaces Properties



$A = \pi ab,\ I_{xx,\,C} = \pi ab^3/4$

(*c*) Ellipse



$A = ab/2,\ I_{xx,\,C} = ab^3/36$

(*d*) Triangle

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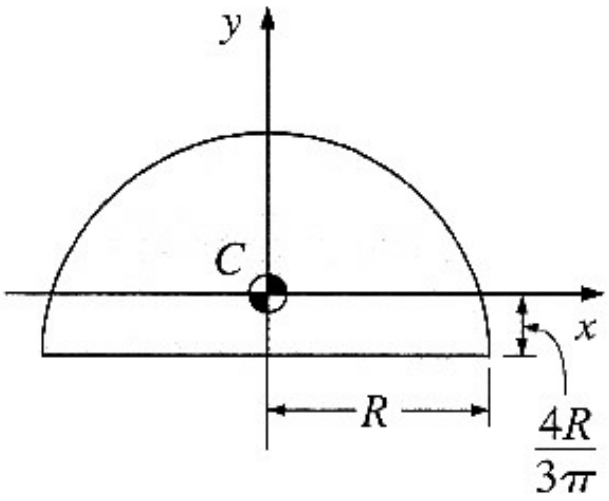
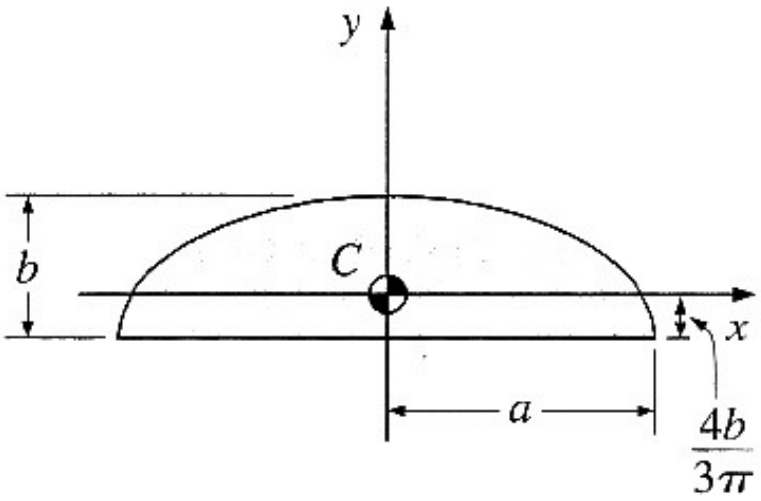
Plane Surfaces Properties



$A = \pi R^2/2,\ I_{xx,\,C} = 0.109757R^4$

(*e*) Semicircle



$A = \pi ab/2,\ I_{xx,\,C} = 0.109757ab^3$

(*f*) Semiellipse

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Problem Set 4

Problem 1

A vertical rectangular plane of height d and base b is submerged in a liquid with its top edge at the liquid surface. Determine the total force F acting on one side and its location from the liquid surface.

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Problem 2

A vertical triangular surface of height d and horizontal base width b is submerged in a liquid with its vertex at the liquid surface. Determine the total force F acting on one side and its location from the liquid surface.

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Problem Set 4

Problem 3

A vertical circular gate of radius r is submerged in a liquid with its top edge flushed on the liquid surface. Determine the magnitude and location of the total force acting on one side of the gate.

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Problem Set 4

Problem 4

A vertical rectangular gate 1.5 m wide and 3 m high is submerged in water with its top edge 2 m below the water surface. Find the total pressure acting on one side of the gate and its location from the bottom.

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Problem Set 4

Problem 5

A vertical circular gate in a tunnel 8 m in diameter has oil ($s = 0.8$) on one side and air on the other side. If soil is 12 m above the invert and the air pressure is 40 kPa, where will a single support be located (above the invert of the tunnel) to hold the gate in position?

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Problem Set 4

Problem 6

Determine the magnitude and location of the total hydrostatic force acting on the 2 m x 4 m gate shown in the figure.